Constraints on the symmetry energy and on neutron skins from the pygmy resonances in ^{68}Ni and ^{132}Sn

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Correlations between the behavior of the nuclear symmetry energy, the neutron skins, and the percentage of energy-weighted sum rule (EWSR) exhausted by the Pygmy Dipole Resonance (PDR) in ^{68}Ni and ^{132}Sn have been investigated by using different Random Phase Approximation (RPA) models for the dipole response, based on a representative set of Skyrme effective forces plus meson-exchange effective Lagrangians. A comparison with the experimental data has allowed us to constrain the value of the derivative of the symmetry energy at saturation. The neutron skin radius is deduced under this constraint.

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One of the interesting problems presently receiving particular attention, is that of the size of the neutron root-mean-square (r.m.s.) radius in neutron-rich nuclei. In fact, this quantity is related to the isospin-dependent part of the nuclear equation of state (EOS) which, in turn, has relevant implications for the description of neutron stars. At present, there is an enormous effort aimed at determining the parameters that govern the asymmetric matter EOS, using both experimental and theoretical tools. Review papers have been devoted to this topic [1, 2].

The energy per particle in a nuclear system characterized by a total density ρ (sum of the neutron and proton densities ρ_n and ρ_p), and by a local asymmetry $\delta \equiv (\rho_n - \rho_p)/\rho$, is usually written as

$$\frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, \delta = 0) + S(\rho)\delta^2. \tag{1}$$

Odd powers of δ are forbidden by the isospin symmetry and the term proportional to δ^4 is found to be negligible. The above equation defines the so-called symmetry energy $S(\rho)$. Determining values of the symmetry energy at various densities of interest for nuclear structure, nuclear reactions, and astrophysics, is one of the great challenges for the physics community.

Information on the symmetry energy can be obtained from various sources, none of them being so far conclusive by itself. A direct correlation between the neutron skin thickness ΔR and the derivative of the symmetry energy at saturation, has been found in Refs. [3, 4]. The derivative of the symmetry energy at saturation is related to the widely used "slope" parameter L by

$$S'(\rho)|_{\rho=\rho_0} = \frac{L}{3\rho_0}. \tag{2}$$

The symmetry energy at saturation, $S(\rho_0)$, is denoted by a_4 or J: we shall use the symbol J in what follows. No measurement of the neutron skin is available which is accurate enough to constrain the slope parameter L. The properties of the isovector Giant Dipole Resonance (IVGDR) [5], of the low-lying electric dipole excitation (the so-called Pygmy Dipole Resonance, PDR) [6], and of the charge-exchange spin-dipole strength [7] have been suggested as constraints. In addition, by means of heavy-ion collisions the symmetry energy has also been probed at subsaturation densities ($0.4 \leq \rho/\rho_0 \leq 1.2$). In Ref. [8], isospin diffusion data from the collision between ^{112}Sn and ^{124}Sn have been analyzed using a transport model in which the momentum-dependent symmetry potential enters as one of the main ingredients. The same data, together with the double ratios of neutron and proton energy spectra, have been analyzed within a different kind of transport model in Ref. [9]. We should also mention that another analysis of isoscaling data has been reported in Ref. [10]. Finally, in the work reported in Ref. [11] a range of values for L is inferred from the analysis of data of radii from antiprotonic atoms.

One of the motivations of the present work lies in the consideration that the values of L extracted from the PDR in ^{132}Sn (between $\approx$ 30 and 60 MeV) [6] and from the GDR in ^{208}Pb [12], are smaller than those deduced from the analysis of the heavy-ion collisions. More precisely, the values of L deduced with the two different approaches overlap only in a small interval (cf., e.g., Fig. 3 of [9]). We would like to pursue in this paper an analysis which is more general than the one performed in Ref. [6], by considering PDRs in two different mass regions and a variety of theoretical models, both nonrelativistic and relativistic. Both classes of mean-field models are successful in describing the nuclear ground states and of many of the excited states (for a review, see Ref. [13]). Our goal is to see whether consistency among different ways of extracting the slope parameter L can be achieved: as a result, one should also expect to be able to better pin down the values of the neutron skin radii.

Progress in the study of the density dependence of the symmetry energy and the neutron radii through the PDR requires more work in two directions. The first is to have more data, particularly for unstable neutron rich-nuclei characterized by a sizeable dipole strength in the low-energy region. The second is a more comprehensive theoretical analysis of the PDRs based on different calculations both of nonrelativistic and relativistic RPA types. In this paper this is realized by taking advantage of a recent experimental datum on the PDR in the neutron-rich ^{68}Ni nucleus [14]. In particular, the present analysis has aimed (i) at finding evidence of a correlation between the values of L and the EWSR exhausted by the PDRs when they are calculated using many Skyrme parameter sets and covariant effective Lagrangians, (ii) at inferring values for the neutron skin radii of ^{68}Ni, ^{132}Sn and ^{208}Pb, and finally (iii) at comparing our deduced value of L with the other ones existing in the recent literature.

The first step consisted in finding correlations between the properties of the PDR and the symmetry energy. Random Phase Approximation (RPA) calculations of the dipole strength have been carried out. Our implementation based on nonrelativistic Skyrme forces is fully self-consistent and discussed, e.g., in Ref. [15]. The Hartree-Fock (HF) equations are solved in a radial mesh extending up to $\approx$4 times the nuclear radius. The continuum is discretized by using box boundary conditions. The model space is large enough so that the well-known double commutator EWSR is exhausted by at least 96%. We have employed 19 different Skyrme sets which can be said to constitute a quite representative ensemble. All of them have an associated value of the nuclear incompressibility K_∞ lying in the interval 210-270 MeV [16]. We do not provide here the original references in which the parameter sets have been introduced: they can be found in [5, 17]. We have checked that our results do not change appreciably if we take out, or add, a few Skyrme parameter sets to our ensemble. The relativistic calculations are based on the well-known relativistic mean field (RMF) theory plus the self-consistent relativistic RPA (RRPA) as described in Refs. [18, 19]. We have employed 7 different parametrizations for the non-linear, meson-exchange effective Lagrangian. In the calculations, box boundary conditions are used: the box radius is set at 30 fm and the radial mesh is 0.1 fm. The model spaces for particle-hole and antiparticle-hole are cut at energy $E_{\rm cutoff}$ = 1039 MeV and -939 MeV, respectively. The references for the seven parameter sets can be found in [20, 21].

We have found a rather good correlation between the parameter L and the percentage of EWSR associated with the PDR. In the theoretical calculations, we consider the whole part of the low-energy region where the strength is not negligible. In the nucleus ^{68}Ni the PDR is associated, as a rule, with a well-defined peak between 9 and 11 MeV, to be compared with the experimental finding of [14], that is, 11 MeV. In few cases the strength is more fragmented and/or at lower energy. We display two typical dipole strength distributions in Fig. 1: the separation between PDR and GDR regions looks quite clear. In the nucleus ^{132}Sn, the peak of the PDR is between 7.5 and 9.5 MeV. The experimental peak energy is 9.8 MeV [6]. The percentages of EWSR are defined in this work with respect to the classical Thomas-Reihe-Kuhn (TRK) value, and vary between 1% and 10%. In general, the relativistic Lagrangians provide larger values for this latter quantity. The PDR energies they provide in the case of ^{132}Sn are also about 1 MeV lower than the experimental value: therefore, trying to constrain the symmetry energy by using the correlation between the PDR energy and the value of S at ρ=0.1 fm^{-3} (plus the experimental datum) was attempted in [20], but it was only possible by means of extrapolation.

In the upper part of Fig. 2 the correlation between the percentage of the EWSR and L is shown for both nuclei ^{68}Ni and ^{132}Sn. The straight lines correspond to linear fits. We have considered the measured values of the EWSR percentage, and deduced a range of acceptable values for L, by taking care both of the experimental error and of the error associated with the fit (the latter being almost negligible with respect to the former). Our results are more general than those presented in Ref. [6] since we consider two different nuclei and many different mean field models. Although we do not include in our analysis all classes of mean field models, we try nonetheless to avoid, as much as we can, possible sources of bias since we avoid restricting to Skyrme sets fitted by the same group with the same protocol. In fact, our sets span a broad range of possible values associated with nuclear matter quantities.

In the case of ^{68}Ni the measured value of the EWSR percentage is 5%$\pm$1.5%. The error includes the uncertainty related to the quantities used to deduce the number from the measurement. It should be noted that the dominating uncertainty (still within 30% of the average value) is that related to the choice of the level density value entering the evaluation of the branching for gamma emission. We have used different level densities obtained by means of either a Shell Model Monte Carlo (SMMC) calculation for this nucleus [22], or global Hartree-Fock-Bogoliubov (HFB) calculations [23, 24]: the largest span goes from 3.5% obtained using [23] to 6.5% using [24].

Our result is that the slope parameter L is constrained to be in the interval 50.3-89.4 MeV or 29.0-82.0 MeV, if we use either the ^{68}Ni results, or the ^{132}Sn results (cf. the lower left panel of Fig. 2). The weighted average, L = 64.8$\pm$15.7 MeV, is displayed in the lower right panel of Fig. 2 (it corresponds to the shaded box). In this panel, the correlation of J and L is provided, so that we can deduce our best value of J which is 32.3$\pm$1.3 MeV. This value is in very good agreement with the value 32.0$\pm$1.8 MeV which is reported in Ref. [6]. The parametrizations of $S(\rho)$ found in Refs. [8, 10] lead to J = 31.6 MeV. Moreover, our result for J overlaps well with the ranges obtained in Refs. [9] (30.2-33.8 MeV) and [25] (31.5-33.5 MeV) (cf. also [26]). From the theoretical point of view, we can consider

very satisfactory that our result for L coincides almost exactly with the value of 66.5 MeV extracted from Bruckner-Hartree-Fock (BHF) calculations in uniform matter that employ realistic two-body and three-body forces [27].

The next step is to use the L value obtained from the PDR computed data points in ^{68}Ni and ^{132}Sn in order to deduce the neutron skin thickness ΔR. First, one can note that the correlation between L and ΔR, when the two quantities are calculated using the models already described, is quite good (cf. Fig. 3). If one imposes the value of L to be in the interval 64.8$\pm$15.7 MeV, one obtains for the skin thickness ΔR=0.200$\pm$0.015 fm for ^{68}Ni, ΔR=0.258$\pm$0.024 fm for ^{132}Sn, and ΔR=0.194$\pm$0.024 fm for ^{208}Pb. These numbers are stable if one tries to constrain them by using the L value from ^{68}Ni only, or ^{132}Sn only, instead of the weighted average. It should also be noted that the values associated with ΔR, both for ^{132}Sn and ^{208}Pb, are in good agreement with the results reported in Ref. [6]. This gives us further confidence on the value of the neutron skin of ^{68}Ni which is determined for the first time through the present analysis. We should recall that the possibility to extract ΔR directly from measurements of the spin-dipole strength has been discussed [28]. For a thorough discussion of the exensive literature appeared in previous decades on this subject we refer the reader to Ref. [29].

In Fig. 4 we show the comparison of the values of L found in our analysis with those found with other analysis and/or other methods. The main point is that the result for L extracted from the PDR in ^{132}Sn is compatible with the one from Ref. [6]; however, combining in our analysis the two PDRs of both ^{132}Sn and ^{68}Ni, we are able to shift the range of L to larger values and to reduce the uncertainty. This solves, to a good extent, the problem that the result from Ref. [6] was not overlapping significantly with the results obtained by the different analysis of heavy-ion collisions. In the lower panel of Fig. 4, one can see that our present finding has a remarkable overlap with the results of most of the other proposed methods to extract L, that involve not only different methodologies but also very different observables. We can conclude that our more general analysis of the extraction of the slope parameter L from the PDR is able to provide a firm result. Another important side result of our work is that we are able for the first time to propose a value for the neutron skin thickness of the neutron-rich ^{68}Ni isotope. More PDR data in other mass regions and/or in long isotopic chains are desirable to increase the predictive power of our procedure. This could lead to determine quite accurately quantities such as the neutron radii, and the parameters governing the density dependence of the symmetry energy, that are fundamental for nuclear physics and for their implications in the study of neutron stars.

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[1] A. W. Steiner *et al.*, Phys. Rep. 411, 325 (2005).
[2] B. A. Li *et al.*, Phys. Rep. 464, 113 (2008).
[3] B. A. Brown, Phys. Rev. Lett. 85, 5296 (2000); S. Typel and B. A. Brown, Phys. Rev. C64, 027302(R) (2001).
[4] R. J. Furnstahl, Nucl. Phys. A706, 85 (2002).
[5] L. Trippa *et al.*, Phys. Rev. C77, 061304(R) (2008).
[6] A. Klimkiewicz *et al.*, Phys. Rev. C76, 051603(R) (2007).
[7] H. Sagawa *et al.*, Phys. Rev. C76, 024301 (2007).
[8] L. W. Chen *et al.*, Phys. Rev. Lett. 94, 032701 (2005).
[9] M. B. Tsang *et al.*, Phys. Rev. Lett. 102, 122701 (2009).
[10] D. V. Shetty *et al.*, Phys. Rev. C76, 024606 (2007).
[11] M. Centelles *et al.*, Phys. Rev. Lett. 102, 122502 (2009); M. Warda *et al.*, Phys. Rev. C80, 024316 (2009).
[12] In [5] the correlation is only between the GDR energy and the symmetry energy at 0.1 fm^{-3}: however, most of the Skyrme forces which have been proposed as acceptable have L below 60 MeV.
[13] M. Bender *et al.*, Rev. Mod. Phys. 75, 121 (2003).
[14] O. Wieland *et al.*, Phys. Rev. Lett. 102, 092502 (2009).
[15] G. Colò *et al.*, Nucl. Phys. A788 (2007) 173c.
[16] S. Shlomo *et al.*, Eur. Phys. J. A30, 23 (2006); G. Colò, Physics of Particles and Nuclei 39, 286 (2008).
[17] J. R. Stone *et al.*, Phys. Rev. C68, 034324 (2003).
[18] P. Ring *et al.*, Nucl. Phys. A694, 249 (2001).
[19] Z. Y. Ma *et al.*, Nucl. Phys. A703, 222 (2002).
[20] Cao Li-gang, Ma Zhong-yu, Chin. Phys. Lett. 25, 1625 (2008).
[21] A. Sulaksono *et al.*, Phys. Rev. C79, 044306 (2009).
[22] Y. Alhassid *et al.*, Phys. Rev. Lett. 99, 162504 (2007); C. N. Gilbreth and Y. Alhassid, private communication.
[23] `www-astro.ulb.ac.be/Html/nld_comb_ph.html`.
[24] `www-nds.ipen.br/RIPL-2/densities.html`.
[25] P. Danielewicz and J. Lee, Nucl. Phys. A818, 36 (2009).
[26] P. Danielewicz, Nucl. Phys. A727, 233 (2003).
[27] I. Vidaña *et al.*, Phys. Rev. C80, 045806 (2009).
[28] A. Krasznahorkay *et al.*, Phys. Rev. Lett. 82, 3216 (1999).
[29] M. N. Harakeh and A. van der Woude, *Giant Resonances* (Clarendon Press, Oxford, 2001).

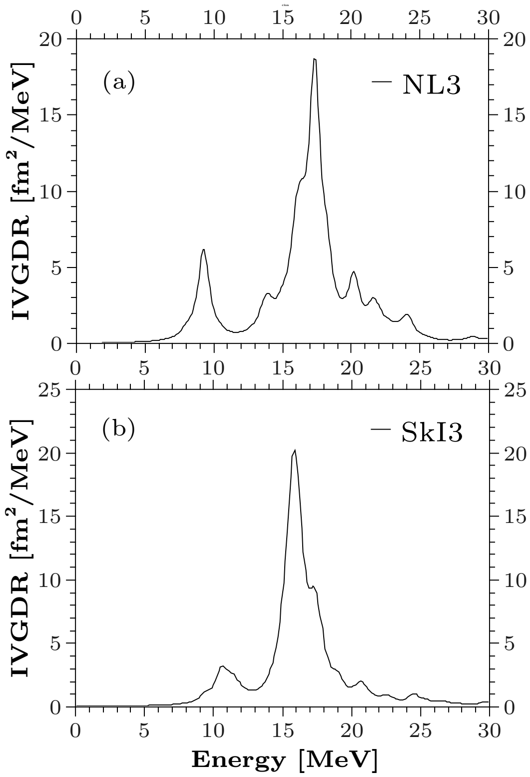

FIG. 1: Two typical dipole strength functions calculated in the nucleus ^{68}Ni. A nonrelativistic and a relativistic example are shown in panels (a) and (b) in which, respectively, the Skyrme force SkI3 and the NL3 parametrization of the effective RMF Lagrangian have been used. The sharp RPA peaks are averaged by using Lorentzian functions having 1 MeV width.

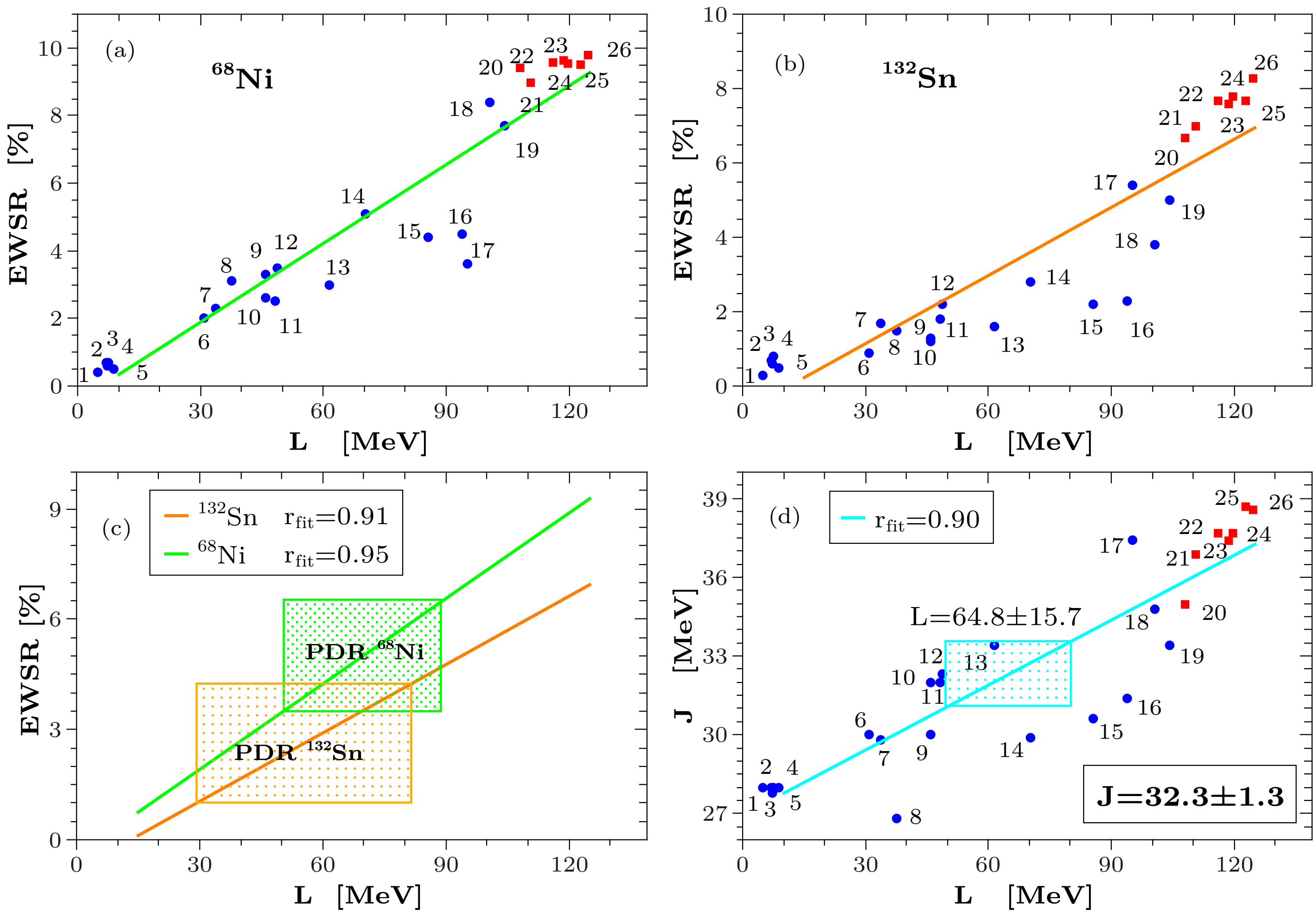

FIG. 2: (Color online) In panels (a) and (b), the correlation between L and the percentage of TRK sum rule exhausted by the PDR, respectively in ^{68}Ni and ^{132}Sn, is displayed. The computed data points are labelled, here and in what follows, by numbers. The correspondence with the parameter sets used is: 1=v090, 2=MSk3, 3=BSk1, 4=v110, 5=v100, 6=SkT6, 7=SkT9, 8=SGII, 9=SkM*, 10=SLy4, 11=SLy5, 12=SLy230a, 13=LNS, 14=SkMP, 15=SkRs, 16=SkGs, 17=SK255, 18=SkI3, 19=SkI2, 20=NLC, 21=TM1, 22=PK1, 23=NL3, 24=NLBA, 25=NL3+, 26=NLE. The straight lines correspond to the results of the fits. In panel (c) we show the same straight lines displayed in (a) and (b), together with the correlation coefficient r and the constraints from experiments [6, 14]. In panel (d) the correlation between L and J is shown. The box corresponds to the value of L deduced from the weighted average of the two values extracted from ^{68}Ni and ^{132}Sn.

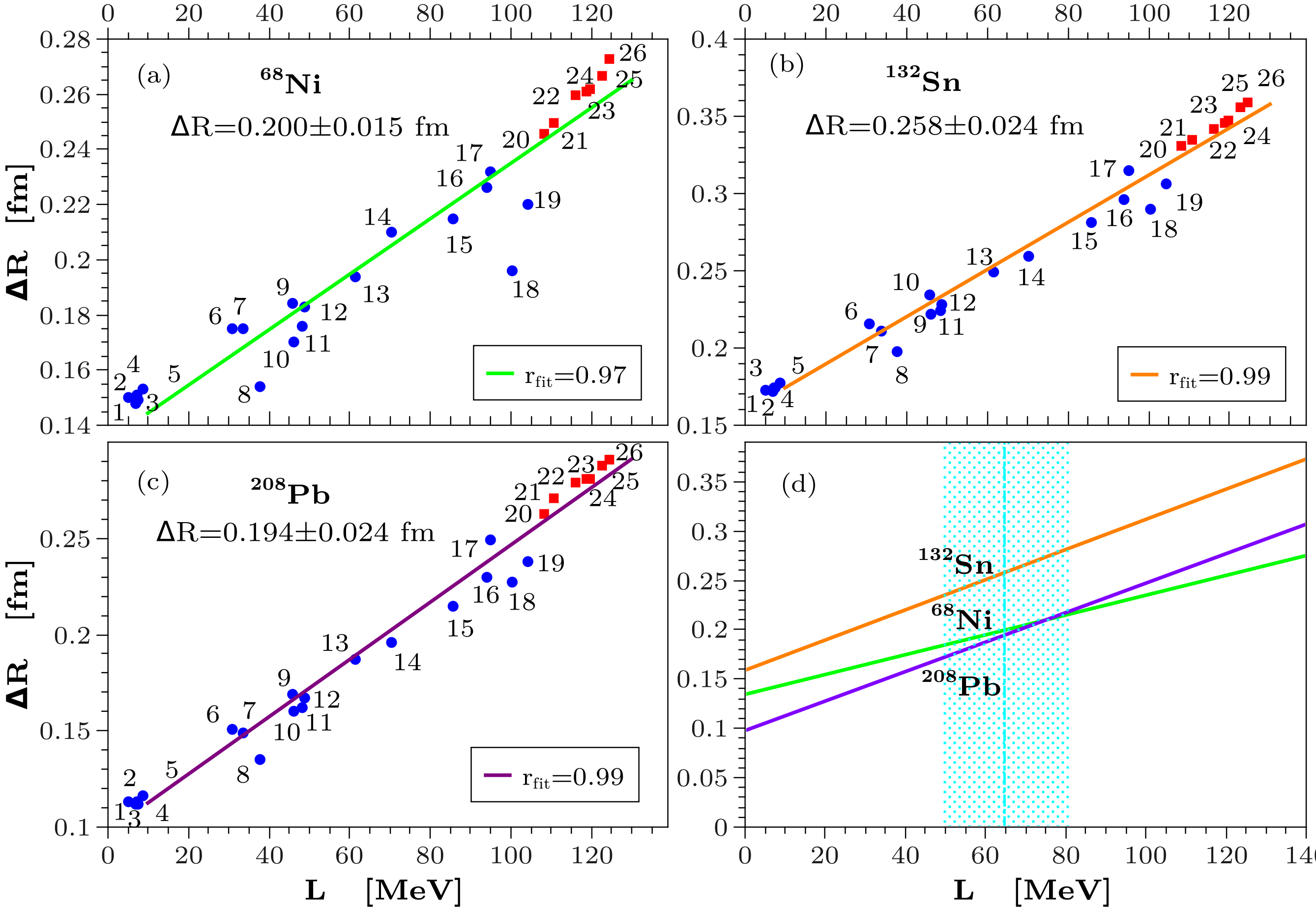

FIG. 3: (Color online) Panels (a), (b) and (c) display the correlations between the neutron skin thickness ΔR and the slope parameter L, in the case of the three nuclei analyzed in this work. The convention is the same as in the previous figure. Under the constraint for L emerging from our analysis [shaded area in panel (d)], the values displayed for the neutron skin thickness in the three nuclei are obtained.

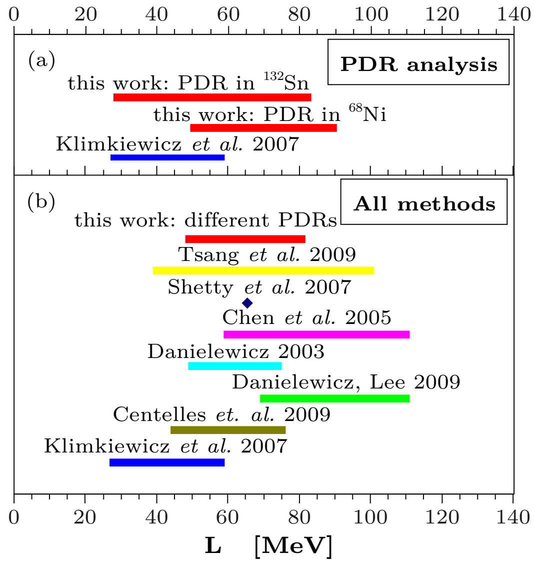

FIG. 4: (Color online) Comparison between the values of L extracted in the present work, and those from the existing literature. In panel (a), we compare our separate results from the PDRs of ^{68}Ni and ^{132}Sn with the result of Klimkiewicz *et al.* [6]. In panel (b), we compare with values extracted from completely different kind of analysis: Tsang *et al.* [9], Shetty *et al.* [10], Chen *et al.* [8], Danielewicz [26], Danielewicz and Lee [25], Centelles *et al.* [11], and Klimkiewicz *et al.* [6].